**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

Sol) Since *μ* = 45 and *σ* = 8

Let x be amount of time taken to complete

Told that the car will be ready in an hour – work began after 10 mins

= 60 – 10 = 50 mins

i.e probability that the service manager can meet his commitment = P(x<=50)

probability that the service manager cannot meet his commitment = 1-P(x<=50)

=1-[(50-45)/8] = 0.625 (from ztable) = 1-0.732 = 0.268.

1. 0.3875
2. 0.2676 Correct
3. 0.5
4. 0.6987
5. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
6. More employees at the processing center are older than 44 than between 38 and 44.

Sol) Given n=400, *μ* = 38, *σ* =6;

As per the question,

P(X>44) = 1- P(X<=44) = 1-stats.norm.cdf(44,38,6) = 0.15

P(38<X<44) = stats.norm.cdf(44,38,6)-1-stats.norm.cdf(38,38,6) = 0.34

Therefore, 0.15 < 0.34, so the given statement is false.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Sol) P(X<30) = stats.norm.cdf(30,38,6) = 0.09.

So, 0.09 of employees = 0.09\*400 = 36.

i.e, 36 employees are expected to be under age of 30; statement is true.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Sol) Since, *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are two independent random variables then

X+Y ~ *N*(μ1+μ2, σ12+ σ22) and X-Y ~ *N*(μ1-μ2, σ12+ σ22)

Similarly, if z=aX+bY, z is linear combination of X and Y, then z ~ *N*(aμ1+bμ2, aσ12+ bσ22)

Therefore, X1+X2 ~ N(2μ, 2σ2)

2X1 = N(2μ, 4σ2)

2X1-(X1+X2) = N(4μ, 6σ2).

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

Sol) μ= 100, σ=20

Probability of value between a and b is 0.99

Probability of value not between a and b is 1-0.99 = 0.01

So, probability towards left from a = -0.01/2 = -0.005

probability towards right from b = 0.01/2 = 0.005

Z = (x – μ)/σ => x = Z\* σ+ μ

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4 = b

Z(0.005)\*20+100 = (-2.57)\*20+100 = 48.6 = a

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5 correct
5. 90.1, 109.9
6. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
7. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

From the empirical rule, 95% of data falls within 2 deviation of the mean = μ ± 2σ

E(X)= E(profit1 + profit2) = 45(5+7) = 540

SD(X) = SD(profit1 + profit2) = √var[(profit1)+var(profit2)] = 45√9+16 = 225

μ ± 2σ = 540 ± 2\*225 => (540-450, 540+450) = (90, 990)

1. Specify the 5th percentile of profit (in Rupees) for the company

To find the 5th percentile we use the formula, μ - 1.5σ = 540 – (1.5\*225) = 202.5.

1. Which of the two divisions has a larger probability of making a loss in a given year?

For division 1 => Z score for the profit of 0 : Z = (x – μ)/σ = (0-5)/3 = -1.66(from ztable) = 0.0485.

For division 2 => Z score for the profit of 0 : Z = (x – μ)/σ = (0-7)/4 = -1.75(from ztable) = 0.0401.

Division 2 has higher probability of making loss.